Infinity and Recursion

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There are some difficulties in the notion of sets, in particular, infinite sets. Among the most serious difficulties is the question about how we can make a general statement about an infinite set. Because there are infinitely many members, we can not do it by checking all the members of it.

Carnap distinguished two kinds of generalities, one of which is what he called 'numerical generality', and the other 'specific generality'. The former is a property which all the members of a set actually have in common, while the latter is one which can be deduced logically from some general property of a set. He contended that a general property of an infinite set must be specific generality. So, he concluded, the general statement about an infinite set is nothing but a statement logically deduced from some statement which is already known to be true for the set.

It is immediately clear that his explanation is not sufficient, for it is difficult to determine what property is specifically general. Moreover, since specific generality is something deduced from another generality, there must be, in the first place, at least one generality known to us which is neither numerical nor specific.

Another question will arise. How can we know a set to be infinite. Illustrating this, I refer to three examples. They are infinite sets given by Dedekind, Zermelo, and Peano. I show that they have a common structure, namely the structure of recursiveness. We can construe these sets as sets of objects which are produced by an endless and recursive procedure. It is this recursive and endless procedure that enable us to reach the notion of infinity.

These things considered, I conclude that as regards infinite sets, there is a generality which I shall call 'conventional generality' given by rules. And this means that infinite sets are artificial, consequently not real objects, in some sense.